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Erratum

Conjunctions, Disjunctions, and Bell-Type Inequalities in Orthoalgebras[1]

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The published version of Theorem 3 does not hold true in general. The mistake in the proof of this theorem consists in unjustified identification of elements $\tilde{a}, \tilde{b}$ that appear in the Mackey decomposition of a pair (a, b) with different, in general, elements $\tilde{a}^*, \tilde{b}^*$ that appear in Mackey decompositions of pairs (a, c) and (a, d) when $c \neq b$ and $d \neq a$.

The correct version of Theorem 3 and its proof is as follows:

Theorem 3. Let L be an orthoalgebra with the UMD property and let $a_1Ca_2Ca_3 \ldots a_nCa_1$, i.e., $a_1, a_2, \ldots, a_n$ be "circularly compatible" elements of L. If p is a state which is dispersion-free on a pair (a_i, a_{i+1}), then the following generalized Bell-type inequality holds:

$$\sum_{\substack{k=1,\ldots,n \\ k\neq i}} S_p(a_k, a_{k+1}) \geq S_p(a_i, a_{i+1}) \tag{7}$$

where we put $a_{n+1} = a_1$.

Proof. Let us note that from the very definitions of the Mackey decomposition, conjunction, and disjunction it follows that for any state p on L

$$p(a) = p(\tilde{a}) + p(a\&b) \tag{8}$$

$$p(b) = p(\tilde{b}) + p(a\&b) \tag{9}$$

and

$$\begin{aligned} p(a|b) + p(a\&b) &= p(\tilde{a} \oplus \tilde{b} \oplus c) + p(c) = p(\tilde{a}) + p(\tilde{b}) + p(c) + p(c) \\ &= p(\tilde{a} \oplus c) + p(\tilde{b} \oplus c) = p(a) + p(b) \end{aligned} \tag{10}$$

Therefore, it follows from (8) and (9) that

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$$\text{if} \quad p(a) = 0, \quad \text{then} \quad p(\tilde{a}) = p(a\&b) = 0 \tag{11}$$

$$\text{if} \quad p(b) = 0, \quad \text{then} \quad p(\tilde{b}) = p(a\&b) = 0 \tag{12}$$

and (10) implies that

$$\text{if } p(a) = p(b) = 1, \quad \text{then} \quad p(a|b) = p(a\&b) = 1. \tag{13}$$

[N.B.: Following the terminology of Pykacz and Santos (1991), we could say that if a pair (a, b) has the unique Mackey decomposition, then any state is a *Jauch–Piron state on* (a, b)]. Finally, let us note that from Lemma 1 it follows that

$$\begin{aligned} \left|p(a) - p(b)\right| &= \left|p(\tilde{a}) + p(a\&b) - p(\tilde{b}) - p(a\&b)]\right| \\ &= \left|p(\tilde{a}) - p(\tilde{b})\right| \leq p(\tilde{a}) + p(\tilde{b}) = S_p(a, b) \end{aligned} \tag{14}$$

Since p is dispersion-free on a pair $(a_i, a_{i+1}$, there are four possibilities: (1) If $p(a_i) = p(a_{i+1}) = 0$, then by (11) and (12), $S_p(a_i, a_{i+1}) = 0$ and (7) is obvious.

(2) If $p(a_i) = 0$ and $p(a_{i+1}) = 1$, then by (11) and (14)

$$\begin{aligned} S_p(a_i, \mathrm{a}_{i+1}) &= 0 + 1 - 2 \cdot 0 = 1 = \left|1 - 0\right| = \left|p(a_{i+1}) - p(a_i)\right| \\ &= \left|p(a_{i+1}) - p(a_{i+2}) + p(a_{i+2}) - p(a_{i+3}) + \cdots \right. \\ &\quad + p(a_n) - p(a_1) + p(a_1) - p(a_2) + \cdots \\ &\quad \left. + p(a_{i-2}) - p(a_{i-1}) + p(a_{i-1}) - p(a_i)\right| \\ &\leq \sum_{\substack{k=1,\ldots,n \\ k \neq i}} \left|p(a_k) - p(a_{k+1})\right| \leq \sum_{\substack{k=1,\ldots,n \\ k \neq i}} S_p(a_k, a_{k+1}) \end{aligned}$$

(3) If $p(a_i) = 1$ and $p(a_{i+1}) = 0$, then we proceed as in case (2).

(4) If $p(a_i) = p(a_{i+1}) = 1$, then $S_p(a_i, a_{i+1}) = 1 + 1 - 2 \cdot 1 = 0$ and (7) is again obvious.

This finishes the proof of Theorem 3.

Of course when Theorem 3 is modified in such a way, the following remarks written just after its original proof are no longer valid: "Let us also note that the assumption made in Theorem 1 that a state p should be dispersion-free on at least one pair of compatible propositions is unnecessary. Therefore, the consequences of Theorem 3 are stronger than those of Theorem 1 since conclusions are not conditioned on the assumption that hypothetical HV states should be dispersion-free on all propositions." However, Theorem 4 remains valid, since in the realm of orthoalgebras it is as straightforward consequence of the correct version of Theorem 3, as Theorem 2 is a consequence of Theorem 1 in the realm of orthomodular posets.